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Scattering of dyons and decay of anti-dyonium

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Abstract Undertaking the study of interaction of dyon with the field of another dyon, we have obtained S-matrix expansion, scattering amplitude and total Hamiltonian for dyon-anti-dyon scattering. Study of bound state of a dyon and an anti-dyon has been carried out and it has been shown that this state is very short lived and decays in to four or six photons depending on the spin-statistics of the dyons involved

Keywords Dyons, anti-dyonium, S-matrix

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Introduction

All current grand unified theories which claim to unify strong, weak and electromagnetic forces, necessarily contain magnetic monopole [1] as their intrinsic part and it is speculated that these theories spearheaded with monopoles and dyons [2,3] (which arises automatically from the semi classical quantization of global charge rotation degree of freedom of magnetic monopole) may be capable of providing answers to various problems left unresolved in standard model. Monopoles which are necessarily dyons [4] have gathered enormous potential importance in connection with the problem of quark confinement [5] magnetic condensation of vacuum and their role in catalyzing proton decay [6] and unification of gravitational and electromagnetic field [7]. Actually, the experimental failure so far to observe monopole and dyon with certainly is itself a significant piece of information, casting doubts on either the standard view of evolution of universe or on the cherished belief about particle physics at extremely short distances. Inspite of enormous potential importance of these particles in all current grand unified theories and in modern high-energy physics, the formalism necessary to describe magnetically charged particle has been clumsy and manifestly non-covariant. The elegant Wu and Yang [8] formalism offered some hope for more convenient treatment of monopoles, but it too appears to be cumbersome for problem involving many monopoles. Brandit and Neri [9] showed that it is extremely difficult to extend Wu and Yang program to the full quantum field theory.

In order to develop a self consistent quantum field theory of generalized electromagnetic field associated with dyons, the authors [10] started with the idea of Cabbibo and Ferrari [11] of two four-potentials to avoid the use of controversial string variables and introduced the generalized charge, generalized four-current and generalized four-potential associated with dyons as complex quantities with their real and imaginary parts are electric and magnetic constituents. We have also undertaken the study of bound states of two dyons and two fermions [12]. Extending this work in the present paper, we have undertaken the study of *S*-matrix expansion for dyon-anti-dyon scattering and obtained the expression for Hamiltonian for the system. We have obtained bound state solutions of a dyon and an anti-dyon in abelian gauge theory and demonstrated that this state is very short lived.

2. S-matrix expansion for dyons

The free field Lagrangian density for a Dirac's particle may be chosen in the following form :

$$L = L\left(\psi,\overline{\psi},\partial_\mu\psi\right) = -\overline{\psi}(x)\left(\gamma_\mu\partial_\mu + m\right)\psi(x) \qquad (2.1)$$

which yields the field equation and its adjoint equation in usual way. When a relativistic Dirac's particle (dyon) interacts with

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generalized electromagnetic field, this interaction is described by the minimal substitution

$$p_\mu \to p'_\mu = p_\mu - |q||V_\mu|$$

or

$$\partial_\mu \to \partial'_\mu = \partial_\mu - i|q||V_\mu|, \qquad (2.2)$$

where q is the charge of dyon defined as

$$q = e - ig, \qquad (2.3)$$

where e is the electric charge and g is the magnetic charge. These are related in the following way :

$$B_\mu = (g/e)A_\mu \qquad (2.4)$$

and V_μ is the potential of generalized electromagnetic field given by

$$V_\mu = A_\mu - iB_\mu, \qquad (2.5)$$

where A_μ and B_μ are electric and magnetic four-potentials of dyon respectively.

With this substitution of (2.2) the Lagrangian density (2.1) of free Dirac's particle is modified to the following form :

$$L(x) = -\overline{\psi}(x)\left[\gamma_\mu\left(\partial_\mu - i|q||V_\mu|\right) + m\right]\psi(x)$$

$$= L_0(x) + L_I(x), \qquad (2.6)$$

where $L_0(x)$ is the free field Lagrangian density given by eq. (2.1) and $L_I(x)$ is the interaction Lagrangian density given by

$$L_I(x) = i\overline{\psi}(x)\gamma_\mu |q||V_\mu|\psi(x), \qquad (2.7)$$

where the generalized four-current density $J_\mu(x)$ is defined as

$$J_\mu(x) = j_\mu(x) - ik_\mu$$

$$= i\left[e\overline{\psi}(x)\gamma_\mu\,\psi(x) - ig\overline{\psi}(x)\gamma_\mu\,\psi(x)\right]$$

$$= iq\,\overline{\psi}(x)\gamma_\mu\,\psi(x), \qquad (2.8)$$

where $j_\mu(x)$ and $k_\mu(x)$ are electric and magnetic four current densities. The interaction Hamiltonian density is given by

$$H_I(x) = -i|q|\overline{\psi}(x)\gamma_\mu |V_\mu|\psi(x) = -i|q|N\left(\overline{\psi}\gamma_\mu |V_\mu|\psi\right). \qquad (2.9)$$

The complete Lagrangian density for a dyon moving in the generalized electromagnetic field of another dyon is given by

$$L(x) = -\overline{\psi}(x)\left[\gamma_\mu\left(\partial_\mu - i|q||V_\mu|\right) + m\right]\psi(x) - \frac{1}{4}G_{\mu\nu}(x)G^{\mu\nu}(x)$$

$$= L_0(x) + L_{Gem}(x) + L_I(x), \qquad (2.10)$$

where $G_{\mu\nu}$, the field tensor associated with generalized electromagnetic field, is given by

$$G_{\mu\nu} = F_{\mu\nu} - F^d_{\mu\nu}, \qquad (2.11)$$

where

$$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} - \frac{1}{2i}\epsilon_{\mu\nu\rho\sigma}\left(B^{\rho,\sigma} - B^{\sigma,\rho}\right).$$

We get the following coupled dyon-photon field equations

$$\left(\gamma_\mu\partial_\mu + m\right)\psi(x) = i|q|\gamma_\mu |V_\mu|\psi(x) \qquad (2.12a)$$

and

$$\partial_\nu G^{\mu\nu}(x) = J^\mu(x). \qquad (2.12b)$$

In order to obtain the perturbative series solutions (i.e. S-matrix expansion), we may write the S-matrix expansion, by choosing the perturbation Hamiltonian $H_I(t)$ in the interaction picture, as

$$S = \sum_n^{\infty} \frac{(-i)^n}{n!}\int dt_1 \int dt_2 \ldots \int dt_n P\left[H_I(t_1)H_I(t_2)\ldots H_I(t_n)\right]$$

where P is the Dyon chronological product and $H_I(t)$ is defined as

$$H_I(t) = \int d\bar{x} H_I(x). \qquad (2.14)$$

$H_I(x)$ in this equation is given by eq. (2.9). Substituting the value of $H_I(t)$ from eq. (2.14) in eq. (2.13), we get

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!}\int d^4x_1 \int d^4x_2 \ldots$$

$$\times \int d^4x_n T\left[H_I(x_1)H_I(x_2)\ldots H_I(x_n)\right], \qquad (2.15)$$

where T denotes the Wick's chronological product. Following the usual process for obtaining various scattering processes we obtain

$$\left[S_3^{(2)}\right]_{fi} = \frac{i|q|^2}{2!}\int d^4x_1 \int d^4x_2 \overline{\psi}^-(x_1)\gamma_\mu\psi^-(x_2)\psi^-$$

$$\times (x_1)\overline{\psi}^+(x_2)\gamma_\nu\left\{D_F(x_1 - x_2)\right\}_{\mu\nu}$$

$$= \frac{i|q|^2}{2!}\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p'_-}}}\bar{u}(p')e^{-ip'-x_1}\right]\gamma_\mu$$

$$\left[\sqrt{\frac{m}{VE_{p'_+}}}u(^-p'_+)e^{-ip'+x_2}\right]\frac{-\delta_{\mu\nu}}{(2\pi)^4}\int\frac{d^4K}{k^2-i\in}e^{iK(x_1-x_2)}\gamma_\nu$$

$$\left[\sqrt{\frac{m}{VE_p}}u(p_-)e^{ip-x_1}\right]\left[\sqrt{\frac{m}{VE_{p_+}}}\bar{u}(-p_+)e^{ip+x_2}\right] \quad (2.16)$$

for dyon-anti-dyon scattering. Carrying out integration over x_1 and x_2, we have

$$\int d^4x_1e^{ix_1(K-p'_-+p_-)}\int d^4x_2e^{ix_2(-K-p'_++p_+)}$$

$$=(2\pi)^4\delta^4(K-p'_-+p_-)(2\pi)^4\delta^4(-K-p'_++p_+). \quad (2.17)$$

Substituting the relation (2.17) in eq. (2.16), we get

$$\left[S_1^{(2)}\right]_{fi} = i(2\pi)^4\delta^4(p_++p_--p'_+-p'_-)$$

$$\left(\frac{m}{VE_{p'_-}}\right)\left(\frac{m}{VE_{p'_+}}\right)\left(\frac{m}{VE_{p_-}}\right)\left(\frac{m}{VE_{p_+}}\right) M_{fi}, \quad (2.18)$$

where Feynman amplitude for the system is given by

$$M_{fi} = -|q|^2\left[\bar{u}(p'_-)\gamma_\mu u(p_-)\right]\left\{D_f(K)\right\}_{\mu\nu}\left[u(^-p_+)\gamma_\nu u(^-p'_+)\right]$$

which may be written as follows for dyon-anti-dyon system ;

$$M_{fi} = -\operatorname{Re}(qq^*)\left[\bar{u}(p'_-)\gamma_\mu u(p_-)\right]D_{\mu\nu}(K)\left[u(-p_+)\gamma_\nu u(-p'_+)\right]$$

$$+\operatorname{Re}(qq^*)\left[\bar{u}(-p_+)\gamma_\mu u(p_-)\right]D_{\mu\nu}(K)\left[\bar{u}(p'_-)\gamma_\nu u(-p'_+)\right], \quad (2.19)$$

where $\operatorname{Re}(qq^*) = e^2+g^2 = |q|^2$,

the first term of this equation corresponds to the scattering process and second to the annihilation process. Let us assume the first term of eq. (2.19) that the two particles are same, with mass m. The subsequent calculations are considerably simplified if the photon propagator $D_{\mu\nu}(K)$ is chosen not in the ordinary gauge but in the Coulomb gauge,

$$D_{00} = -\frac{4\pi}{K^2}, D_{0i} = 0, D_{ik} = \frac{4\pi}{K^2-\omega^2/c^2}\left(\delta_{ik}-\frac{K_iK_k}{K^2}\right). \quad (2.20)$$

Then the first term of the scattering amplitude [eq. (2.19)] is

$$M_{fi}^{(scatt)} = -\operatorname{Re}(qq^*)\left[(\bar{u}'_-\gamma_0u_-)(\bar{u}_+\gamma_0u'_+)D_{00}\right.$$

$$\left.+(\bar{u}'_-\gamma_iu_-)(\bar{u}_+\gamma_ku'_+)D_{ik}\right]. \quad (2.21)$$

If all terms in $1/c$ are neglected, the second term in the braces vanishes, and the first term gives

$$M_{fi}^{(scatt)} = -4m^2\left(w'^{(0)*}_-w^{(0)}_-\right)\left(w^{(0)*}_+w'^{(0)}_+\right)U(K), \quad (2.22)$$

where

$$U(K) = -\frac{4\pi\operatorname{Re}(qq^*)}{K^2} \quad (2.23)$$

and $w_-^{(0)}$, $w_+^{(0)}$ denote the spinor (two components) amplitudes of the non-relativistic plane waves.

In the next approximation (with respect to $1/c$), the Schrödinger wave function of the free-particle ϕ_{Sch} (normalized by the integral $\int|\phi_{Sch}|^2d^3x$) satisfies the equation

$$H^{(0)}\phi_{Sch} = (\varepsilon-mc^2)\phi_{Sch}$$

$$H^{(0)} = \frac{p^2}{2m}-\frac{p^4}{8m^3c^2}, p = -i\hbar\nabla \quad (2.24)$$

which includes the next term in the expansion of the relativistic expression for the kinetic energy. The (spinor) amplitude of this plane wave will be denoted w, which tend to $w^{(0)}$ as $1/c \to 0$.

The bispinor amplitude u of the free particle can be expressed in terms of Schrödinger amplitude w, with sufficient accuracy, by [15]

$$u = \sqrt{(2m)}\begin{pmatrix}\left(1-\frac{p^2}{8m^2c^2}\right)w\\ \left(\sigma\cdot p/2mc\right)w\end{pmatrix} \quad (2.25)$$

This formula gives

$$\bar{u}'_-\gamma_0u_- = u'^*_-u_- = 2m\left(1-\frac{p'^2_-+p^2_-}{8m^2c^2}\right)w'^*_-w_-$$

$$+\frac{1}{2mc^2}w'^*_-(\sigma\cdot p'_-)(\sigma\cdot p_-)w_-$$

$$= 2mw'^*_-\left(1-\frac{K^2}{8m^2c^2}+\frac{i\sigma\cdot K\times p_-}{4m^2c^2}\right)w_-,$$

$$\bar{u}'_-\gamma u_- = u'^*_-\alpha u_- = \left(1/c\right)w'^*_-\left[\sigma(\sigma\cdot p_-)+(\sigma\cdot p'_-)\sigma\right]w_-$$

$$= \left(1/c\right)w'^*_-\left[i\sigma\times K+2p_-+K\right]w_-,$$

where $K = p'_- - p_- = p_+ - p'_+$. The corresponding expressions for $(\bar{u}_+\gamma_0u'_+)$ and $(\bar{u}_+\gamma u'_+)$ differ in that the suffix – is replaced by + and K by $-K$. We now substitute these expressions in eq. (2.21). Since the product $(\bar{u}'_- \gamma u_-)(\bar{u}_+ \gamma u'_+)$ already contains the factor $1/c^2$, the term ω^2/c^2 in the denominator of D_{ik} may be neglected. The $M_{fi}^{(scatt)}$ is then

$$M_{fi}^{(scatt)} = -4m^2\left[w'^*_- w^*_+ U(p_-, p_+, K)\, w_- w'_+\right] \quad (2.26)$$

where

$$U(p_-, p_+, K) = -4\pi\,\mathrm{Re}(qq^*)\Big[\frac{1}{K^2} - \frac{1}{4m^2c^2} + \frac{(K\cdot p_-)(K\cdot p_+)}{m^2K^4}$$

$$-\frac{p_- \cdot p_+}{m^2K^2} + \frac{i\sigma_- \cdot K\times p_-}{4m^2c^2K^2} - \frac{i\sigma_- \cdot K\times p_+}{2m^2c^2K^2} + \frac{i\sigma_+ \cdot K\times p_+}{4m^2c^2K^2}$$

$$+\frac{i\sigma_+ \cdot K\times p_-}{2m^2c^2K^2} + \frac{(\sigma_- \cdot K)(\sigma_+ \cdot K)}{4m^2c^2K^2} - \frac{\sigma_- \cdot \sigma_+}{4m^2c^2}\Big]; \quad (2.27)$$

the suffix – and + to the Pauli matrices indicate the spinor indices on which they act, σ_- act on w_- and σ_+ on w_+.

The function $U(p_-, p_+, K)$ is the particle interaction operator in the momentum representation. We obtain

$$U(p_-, p_+, r) = -\frac{\mathrm{Re}(qq^*)}{m^2c^2} + \frac{\mathrm{Re}(qq^*)\pi\hbar^2}{m^2c^2}\delta(r) + \frac{\mathrm{Re}(qq^*)}{2m^2c^2r}\left[p_- \cdot p_+ + \frac{r.(r.p_-)p_+}{r^2}\right]$$

$$+\frac{\mathrm{Re}(qq^*)\hbar}{4m^2c^2r^3} r\times p_- \cdot \sigma_- - \frac{\mathrm{Re}(qq^*)}{4m^2c^2r^3} r\times p_+ \cdot \sigma_+$$

$$+\frac{\mathrm{Re}(qq^*)\hbar}{2m^2c^2r^3}\left[r\times p_- \cdot \sigma_+ - r\times p_+ \cdot \sigma_-\right] - \frac{\mathrm{Re}(qq^*)\hbar^2}{4m^2c^2}$$

$$\left[\frac{\sigma_- \cdot \sigma_+}{r^3} - 3\frac{(\sigma_- \cdot r)(\sigma_+ \cdot r)}{r^5} - \frac{8\pi}{3}\sigma_- \cdot \sigma_+ \delta(r)\right] \quad (2.28)$$

in coordinate representation by the usual techniques.

The total Hamiltonian of the two particle system in this approximation is

$$H = H_-^{(0)} + H_+^{(0)} + U(p_-, p_+, r), \quad (2.29)$$

where $H^{(0)} = H_-^{(0)} + H_+^{(0)}$ is the free particle Hamiltonian and eq. (2.24) is given as

$$H^{(0)} = \frac{1}{2m}\left(p_-^2 + p_+^2\right) - \frac{1}{8m^3c^2}\left(p_-^4 + p_+^4\right). \quad (2.30)$$

Let us now consider the transformation of the second term of the eq. (2.19). Here, we use the photon propagator in the ordinary gauge ;

$$D_{\mu\nu} = \frac{4\pi}{k^2} g_{\mu\nu} = \frac{4\pi}{\frac{\omega^2}{c^2} - k^2} g_{\mu\nu}.$$

In the present case $k = p_+ + p_-$ and since the particles are almost non-relativistic, we have

$$\frac{\omega^2}{c^2} \equiv \frac{(\varepsilon_+ + \varepsilon_-)^2}{c^2} \approx 4m^2c^2 \gg (p_+ + p_-)^2 \equiv k^2. \quad (2.31)$$

For photon propagator it is therefore sufficient to write

$$D_{\mu\nu} \approx \left(\frac{\pi}{m^2c^2}\right) g_{\mu\nu}.$$

The amplitude $u(p)$ in the zero-order approximation is given as follows ;

$$u(-p_-) = \sqrt{(2m)}\begin{pmatrix} w_-^{(0)} \\ 0 \end{pmatrix}, u(-p_+) = \sqrt{(2m)}\begin{pmatrix} 0 \\ w'^{(0)} \end{pmatrix}$$

where $w_-^{(0)}, w'^{(0)}$ are the three dimensional spinors, the index (0) will henceforward be omitted. With these amplitudes we have

$$\bar{u}(-p_+)\gamma_0 u(p_-) = u^*(-p_+)u(p_-) = 0,$$

$$\bar{u}(-p_+)\gamma u(p_-) = u^*(-p_+)\alpha u(p_-) = 2m(w^*\sigma w_-)$$

On substuting of these expressions, the annihilation term in the scattering amplitude (2.19) becomes

$$M_{fi}^{(ann)} = -\frac{\mathrm{Re}(qq^*)\pi}{m^2c^2}(2m)^2(w^*\sigma w_-)(w'^*_- \sigma w') \quad (2.32)$$

The anti-dyon amplitudes are got from $u(-p_+)$ by charge conjugtion and corresponding spinors (which are denoted by w_+) are related to w by

$$w_+ = \sigma_y w^*, \text{ whence}$$

$$w^* = \sigma_y w_+ = -w_+\sigma_y, w = -\sigma_y w_+^* \quad (2.33)$$

Secondly, the scattering amplitude must be brought to a form in which the dyon spinors (w_- and w'_-) are contracted and likewise the anti-dyon spinors (w_+ and w'_+). This is achieved by means of the formula

$$(w^*\sigma w_-)(w'^*_- \sigma w') = \frac{1}{2}(w'^*_- w_-)(w^* w')$$

$$-\frac{1}{2}(w'^*_- \sigma w_-)(w^*\sigma w'). \quad (2.34)$$

Finally, expressing w and w' in terms of w_+ and w'_+ by (2.33) we get

$$(w^* w') = (w'^*_+ w_+)$$

$$\left(w^*\sigma w'\right) = -\left(w_+'^*\sigma w_+\right). \tag{2.35}$$

Substituting eq. (2.35) in (2.34) and then in eq. (2.32), we obtain the final expression for the annihilation part of the scattering amplitude;

$$M_{fi}^{(ann)} =$$

$$-4m^2 \; w_-'^* w_+'^* \left[\frac{\pi\,\mathrm{Re}(qq^*)}{2m^2c^2}(3+\sigma_+\cdot\sigma_-)\right] w_- w_+ \Big). \tag{2.36}$$

The expression in the square brackets is the interaction operator in the momentum representation. The corresponding coordinate operator is

$$U^{(ann)}(r) = \frac{\pi\hbar^2\,\mathrm{Re}(qq^*)}{2m^2c^2}(3+\sigma_+\cdot\sigma_-)\delta(r), \tag{2.37}$$

where

$$r = r_- - r_+ .$$

The total dyon-anti-dyon interaction operator is $U + U^{(ann)}$, with U and $U^{(ann)}$ given by eqs. (2.28) and (2.37) respectively. Therefore,

$$U + U^{(ann)} = -\frac{(e^2+g^2)}{r} + \frac{(e^2+g^2)\pi\hbar^2}{m^2c^2}\delta(r)$$

$$\frac{(e^2+g^2)}{2m^2c^2r}\left[p_-\cdot p_+ + \frac{r\cdot(r\cdot p_-)p_+}{r^2}\right]$$

$$+\frac{(e^2+g^2)\hbar}{4m^2c^2r^3} r\times p_-\cdot\sigma_- - \frac{(e^2+g^2)}{4m^2c^2r^3} r\times p_+\cdot\sigma_+$$

$$+\frac{(e^2+g^2)\hbar}{2m^2c^2r^3}\left[r\times p_-\ \sigma_+ - r\times p_+\cdot\sigma_-\right]$$

$$\frac{(e^2+g^2)\hbar^2}{4m^2c^2}\left[\frac{\sigma_-\cdot\sigma_+}{r^3} - 3\frac{(\sigma_-\cdot r)(\sigma_+\cdot r)}{}\right.$$

$$\left.\frac{8\pi}{3}\sigma_-\cdot\sigma_+\delta(r) - 2\pi(3+\sigma_+\cdot\sigma_-)\delta(r)\right|. \tag{2.38}$$

3. Bound state of a dyon and an anti-dyon in abelian gauge theory

The results obtained in the previous section can be applied to anti-dyonium (*i.e.* bound state of a dyon and an anti-dyon). In the centre-of-mass system, the dyon and anti-dyon momentum operators in anti-dyonium are $p_- = -p_+ \equiv p$, where $p = -i\hbar\nabla$ is the operator of the momentum of relative motion corresponding to the relative position vector $r = r_- - r_+$. The total Hamiltonian for the system is

$$H = \frac{p^2}{m} - \frac{(e^2+g^2)}{r} + V_1 + V_2 + V_3 ,$$

$$V_1 = -\frac{p^4}{4m^3c^2} + \frac{(e^2+g^2)\pi\hbar^2}{m^2c^2}\delta(r)$$

$$-\frac{(e^2+g^2)}{2m^3c^2r}\left[p^2 + \frac{r\cdot(r\cdot p)p}{r^2}\right],$$

$$V_2 = \frac{3(e^2+g^2)\pi\hbar^2}{2m^2c^2r^3} l\cdot s,$$

$$V_3 = \frac{3(e^2+g^2)\pi\hbar^2}{}\left[\frac{(s\cdot r)(s\cdot r)}{} - \frac{1}{}s^2\right]$$

$$\frac{(e^2+g^2)\pi\hbar^2}{m^2c^2}\left(\frac{7}{3}s^2 \quad 2\right|\delta(r). \tag{3.1}$$

Here $\hbar l = r\times p$ is the orbital angular momentum operator, $s = \frac{1}{2}(\sigma_+ + \sigma_-)$ is the total spin operator of the system, square of which is $s^2 = \frac{1}{2}(3+\sigma_+\cdot\sigma_-)$. V_1 includes all the purely orbital correction terms, V_2 includes the spin-orbit interaction and V_3 includes the spin-spin and annihilation interactions.

The unperturbed Hamiltonian

$$H^{(0)} = \frac{p^2}{} - \frac{(e^2+g^2)}{} \tag{3.2}$$

naturally differs from the Hamiltonian of dyonium [13] only in that the mass of the dyon is replaced by the reduced mass $\frac{1}{2}m$. The energy levels of anti-dyonium therefore, have absolute values which are half of those for dyonium [13]

$$E = -\frac{m(e^2+g^2)^2}{4\hbar^2n^2} \tag{3.3}$$

where n is the principal quantum number. The remaining terms in eq. (3.1) cause a splitting of the levels of eq. (3.3). The resulting levels are classified primarily by the values of the total angular momentum J. We also observe that particle spin operator appear in Hamiltonian (3.1) only through the sum s. This means that the Hamiltonian commutes with the squared total spin operator s^2, *i.e.* the values of the total spin continues to be conserved in the approximation considered. The energy levels of anti-dyonium can therefore be classified by the total spin, which takes the values $s = 0$ and $s = 1$. The levels with spin-o are called para-anti-dyonium levels, and those with spin-1 are ortho-anti-dyonium levels. The orbital magnetic moment of anti-dyonium is always zero, since for it $r_+\times p_+ = r_-\times p_-$ and we have the operator

$$\mu_1 = \frac{(e^2+g^2)^{1/2}}{2mc}(r_+\times p_+ - r_-\times p_-) = 0. \tag{3.4}$$

The spin magnetic moment operator,

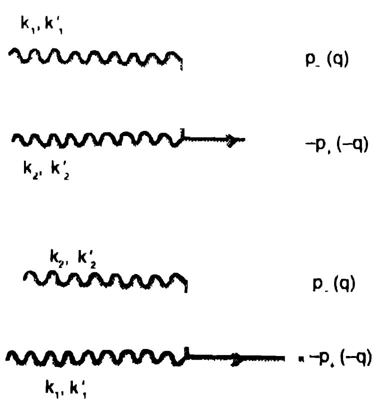
$$\mu_s = \frac{(e^2+g^2)^{1/2}}{2mc}(\sigma_+ - \sigma_-) \tag{3.5}$$

is not proportional to the total spin operator $s = \frac{1}{2}(\sigma_+ + \sigma_-)$, and the operator s^2 and μ_s^2 do not commute. The splitting in energy (fine structure) is given by the mean value of correction terms in the Hamiltonian (3.1), calculated by means of the wave functions of the unperturbed states with different values of $J = l$ ($l = 0, 1, \ldots, (n-1)$). When $s = 0$, the only non-zero contributions come from V_1 and the second term in V_3. We have obtained the required energy levels of para-anti-dyonium as

$$E_{nl} = -\frac{1}{4n^2} - \alpha^2 \frac{m(e^2+g^2)^2}{\hbar^2}\frac{1}{2n^3}\left(\frac{1}{2l+1} - \frac{11}{32n}\right) \tag{3.6}$$

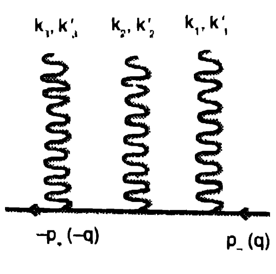
where α is the fine structure constant for the system, given by $(e^2+g^2)/4\pi\epsilon_0 \hbar c$.

The main processes, which determines the lifetime of anti-dyonium, are therefore four-photon decay for para-anti-dyonium and six-photon decay for ortho-anti-dyonium. The annihilation of a dyon and an anti-dyon (with four momenta p_- and p_+) to form four-photon (two of them corresponding to electric charge of the system having momenta K_1 and K_2 and two-photon corresponding to magnetic charge of the system which are extraordinarily energetic having momenta K'_1 and K'_2) [14,15]. This is shown in Figure 1. Similarly, for ortho-anti-dyonium decay form six-photon (three of them corresponding to electric charge of the system having momenta K_1, K_2 and K_3 and three-photon corresponding to magnetic charge of the system which are extraordinarily energetic having momenta K'_1, K'_2 and K'_3). This process is shown in Figure 2.



(a) Four photons decay

Figure 1.



(b) Six photons decay

Figure 2.

Solid black wavy lines denote ordinary photons corresponding to electric charge Grey wavy lines denote extra ordinarily energetic photons corresponding to magnetic charge.

In these figures K_1, K_2 and K_3 are the momenta of photons corresponding to electric charge and K'_1, K'_2 and K'_3 are the momenta of photons corresponding to magnetic charge

Let $\bar{\sigma}_{4\gamma}$ be the cross section for four-photon decay of a free pair average over the spin directions of both particles. In the non-relativistic limit

$$\sigma_{4\gamma} = \frac{\pi e^4}{m^2c^4}\frac{c}{v} + \frac{\pi g^4}{m^2c^4}\frac{c}{v}, \tag{3.7}$$

where v is the relative velocity of the particles. The annihilation probability $\bar{w}_{4\gamma}$ is obtained on multiplying $\bar{\sigma}_{4\gamma}$ by the flux density $v|\psi(0)|^2$. The normalized wave function $\psi(r)$, of anti-dyonium ground state is given as

$$\psi(r) = \frac{1}{(\pi a^3)^{1/2}} e^{-r/a}, \quad a = \frac{2\hbar^2}{m(e^2+g^2)}, \tag{3.8}$$

where a is the Bohr radius of anti-dyonium and m is the mass of dyon. The mean decay probability $\bar{w}_{4\gamma}$ is related to the para anti-dyonium decay probability w_0 in the following manner

$$\bar{w}_{4\gamma} = \frac{1}{4} w_0 \,, \tag{3.9}$$

so

$$w_0 = 4|\psi(0)|^2 \left(v\bar{\sigma}_{4\gamma}\right)_{v\to 0}$$

With the help of eqs. (3.7), (3.8) and (3.10), we get

$$w_0 = \frac{m\left(e^{10} + e^4g^6 + 3e^6g^4 + 3e^8g^2\right)}{2\hbar^6c^3} + \frac{m\left(g^{10} + g^4e^6 + 3g^8e^2 + 3g^6e^4\right)}{2\hbar^6c^3} \tag{3.11}$$

Thus, the lifetime of para-anti-dyonium is

$$\tau_0 = \frac{2\hbar}{mc^2}\left[\frac{1}{\alpha_e^5} + \frac{1}{\alpha_g^5}\right] = 2.648963 \times 10^{-14} \text{ seconds} \tag{3.12}$$

This process is shown in Figure 1. Similarly, we can show that the decay probability of ortho-anti-dyonium, related to the spin averaged cross section for six-photon decay of a free pair, is given by

$$w_1 = \frac{4}{3}\bar{w}_{6\gamma} = \frac{4}{3}|\psi(0)|^2 \left(v\bar{\sigma}_{6\gamma}\right)_{v\to 0}. \tag{3.13}$$

The statistical weight of the spin-1 being 3/4. As such, we may mention that

$$\bar{\sigma}_{6\gamma} = \frac{4(\pi^2-9)c\alpha_e e^4}{3vm^2c^4} + \frac{4(\pi^2-9)c\alpha_g g^4}{3vm^2c^4}. \tag{3.14}$$

Substituting the value of $\bar{\sigma}_{6\gamma}$ in eq. (3.13) and using eq. (3.8), we get

$$w_1 = \frac{2m\left(e^2+g^2\right)^3\left(\pi^2-9\right)\alpha_e e^4}{9\pi\hbar^6c^3}$$

$$\frac{2m\left(e^2+g^2\right)^3\left(\pi^2-9\right)\alpha_g g^4}{9\pi\hbar^6c^3} \tag{3.15}$$

The lifetime of ortho-anti-dyonium is therefore

$$\frac{9\pi\hbar}{2mc^2(\pi^2-9)}\;\frac{1}{\alpha_e^6}+\frac{1}{\alpha_g^6} \approx 2.9866\times10^{-11}\ \text{seconds} \tag{3.16}$$

which can be represented diagramatically as shown in Figure 2.

4. Conclusion

Eq (2.10) is the Lagrangian density for a dyon moving in the generalized electromagnetic field (of another dyon) which leads to the coupled field eqs. (2.12a) and (2.12b). Eq. (2.15) is the S matrix expansion and with the help of this equation we have obtained scattering amplitude for dyon-anti-dyon system defined by eq. (2.19). From eq. (2.19), we have obtained the interaction operator ($U + U^{(ann)}$) described by eq. (2.38). Eq. (3.1) is the Hamiltonian for anti-dyonium, eq. (3.6) is the energy levels of para-antidyonium and eq. (3.7) is the cross section for four-photon decay. Eq. (3.12) describes the lifetime of para-anti-dyonium which is 2.648963×10^{-14} seconds. This lifetime is very small in comparison to the lifetime of para positronium (1.23×10^{-10} seconds). Eq. (3.16) describes the lifetime of ortho-anti-dyonium (2.9866×10^{-11} seconds) is also very small in comparison to the lifetime of ortho-positronium (1.4×10^{-7} seconds). It implies that this bound state is very short- lived and it decays in to two modes. For para-anti-dyonium, we get almost instantly four-photon and for ortho-anti-dyonium, we get six-photon, three of which must be highly energetic.

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